Hard Encoding of Physics for Learning Spatiotemporal Dynamics

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Spatiotemporal Data-driven Modeling

- A dynamical system $u_t = F(x, t, u, u^2, \nabla u, \nabla^2 u, \ldots)$ unknown

**Given:**
- Low-resolution and noisy measurements;
- Optionally, prior knowledge on the system.

**Objectives:**
- To establish a data-driven model that gives high-resolution (spatially and temporally) prediction;
- The data-driven model generalizes well.

Boundary Encoding

- BCs (if available) are forcibly encoded to ensure solution accuracy

Element-wise Product Layer

- Π-block approximation

$$F(u) = \sum_{c=1}^{N_c} f_c \prod_{l=1}^{N_l} D^{(C_l)} \odot u$$

- Benefits
  - Multiplicative form makes the learned model more interpretable;
  - Enables a better approximation to nonlinear terms like $uu_x$ and $u \cdot \nabla u$.

Physics-encoded Recurrent Conv Neural Network (PeRCNN)

**Network architecture**

- Initial state generator (ISG)
- Recurrent block (II-block)

**Components**
- ISG: to generate high-res initial condition;
- II-block: recurrent block to update state variables.

**Design intuitions**
- Residual connection mimics forward Euler scheme;
- Physics-based Conv layer encodes existing terms in $F$;
- Element-wise product layer approximates nonlinear terms better.

Numerical Experiments

- **Baselines:** ConvLSTM, Deep Hidden Physics Model (DHPM) and Recurrent ResNet

- PeRCNN outperforms the baselines on accuracy;
- PeRCNN generalizes well beyond the training region where no data is available.

References