Hard Encoding of Physics for Learning Spatiotemporal Dynamics

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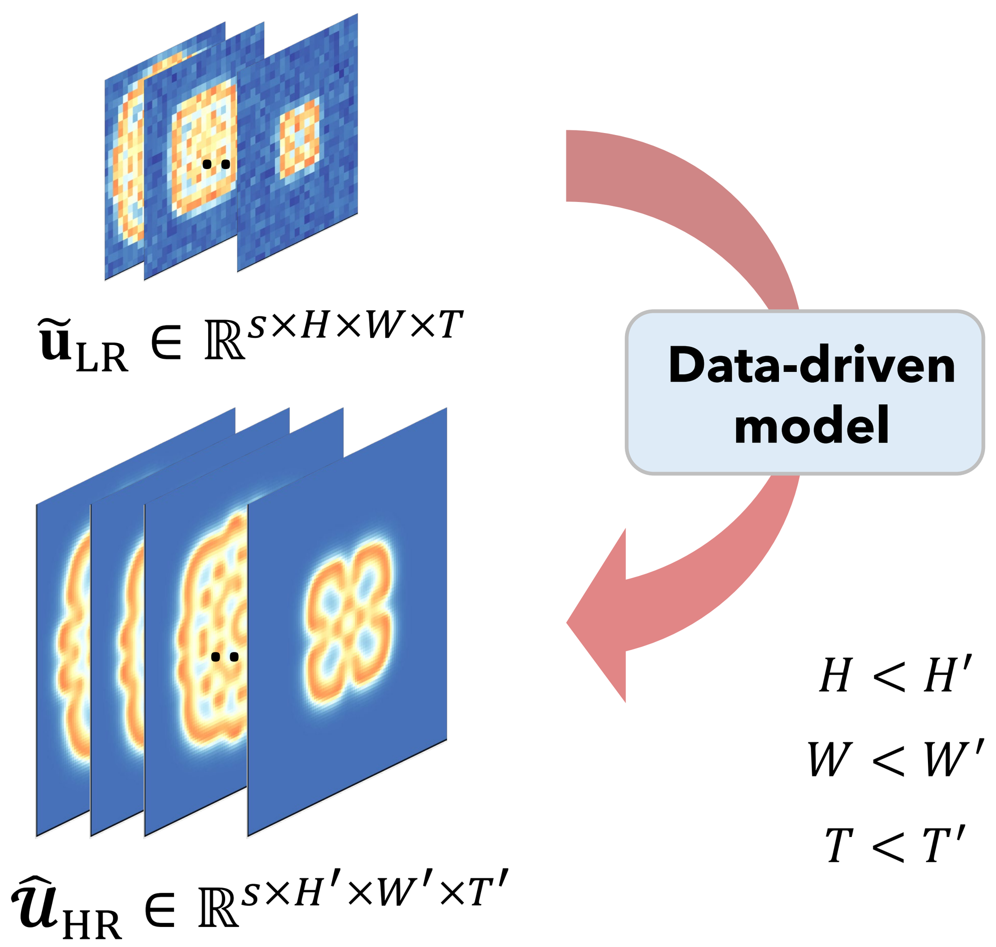
Spatiotemporal Data-driven Modeling

- **A dynamical system** $\mathbf{u}_t = \mathcal{F}\left(\mathbf{x}, t, \mathbf{u}, \mathbf{u}^2, \nabla\mathbf{u}, \nabla^2\mathbf{u}, \dots\right)$ ***unknown***

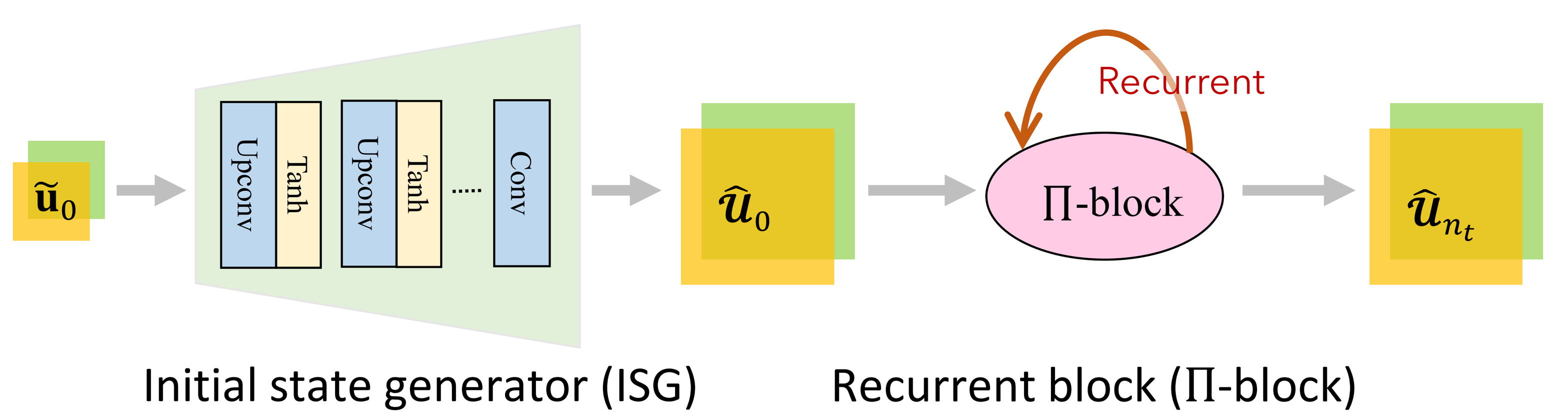
Given:
- **Low-resolution** and **noisy** measurements;
- Optionally, prior knowledge on the system.

Objectives:
- To establish a data-driven model that gives **high-resolution** (spatially and temporally) prediction;
- The data-driven model **generalizes** well.

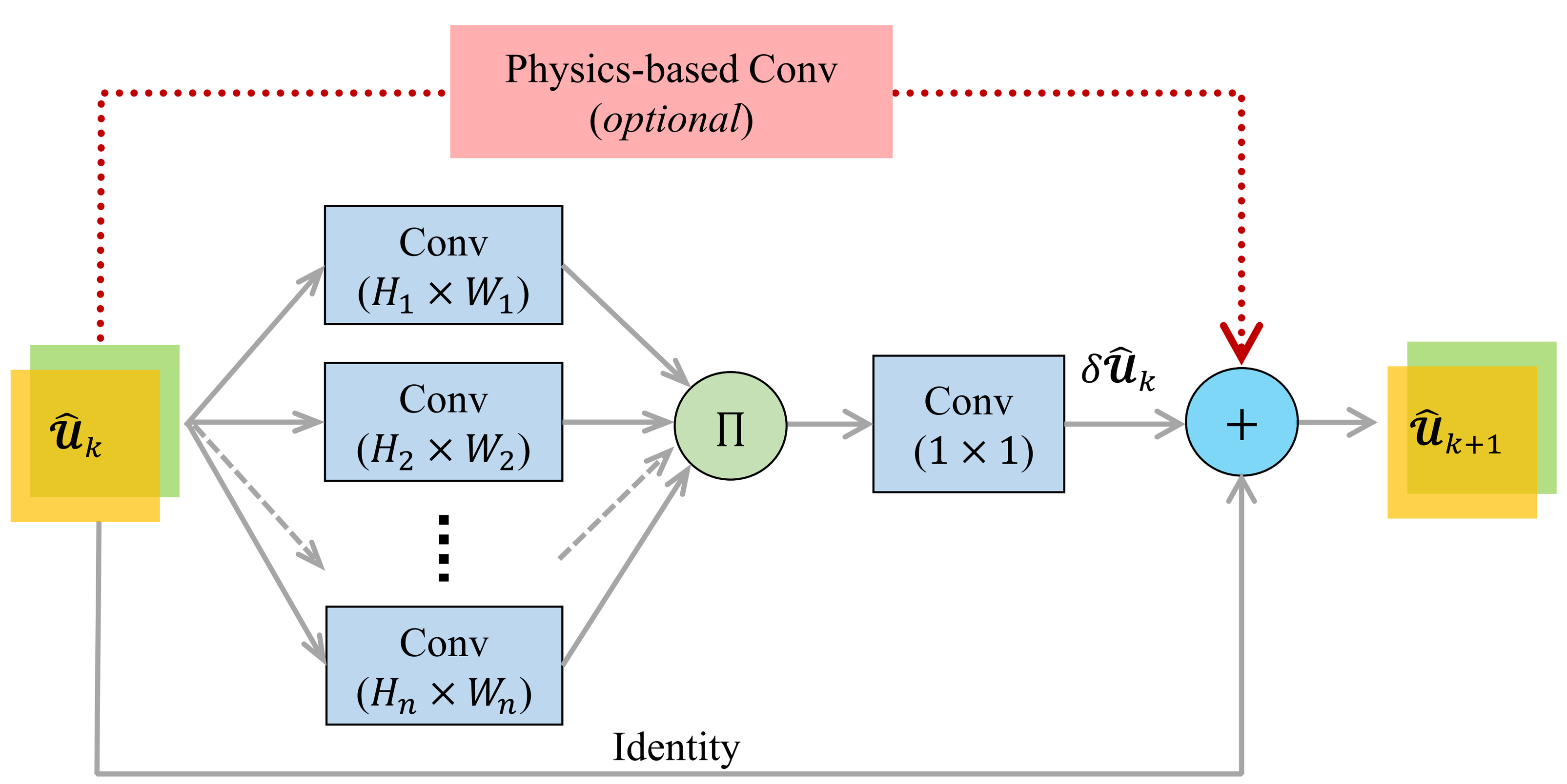


Physics-encoded Recurrent Conv Neural Network (PeRCNN)

- **Network architecture**



- **Details of Π-block**

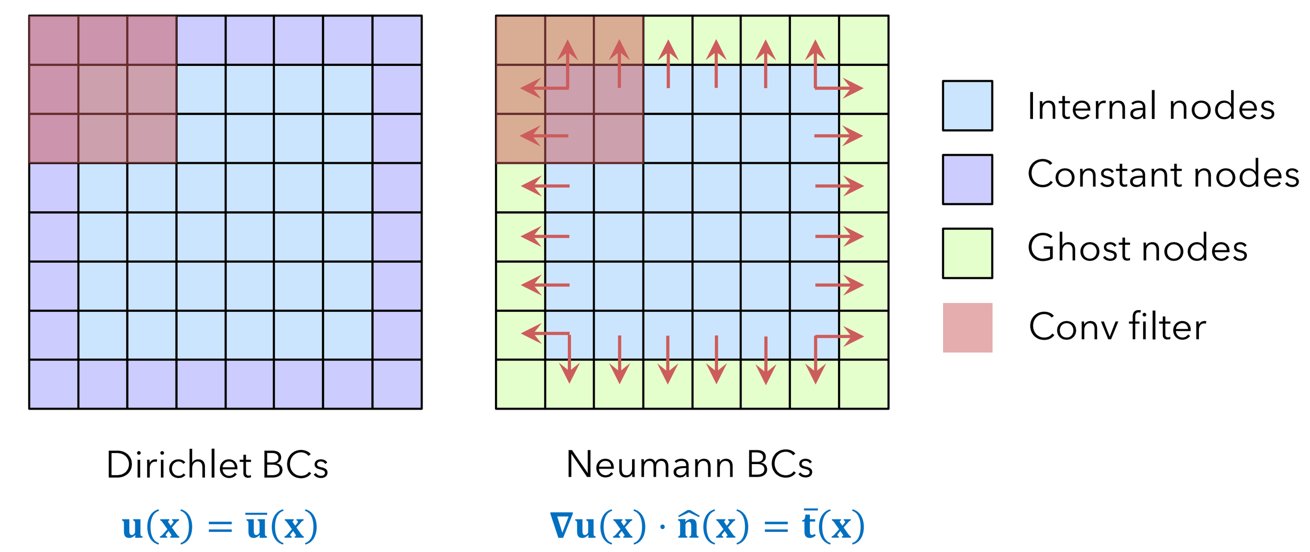


- **Components**
  - **ISG**: to generate high-res initial condition;
  - **Π-block**: recurrent block to update state variables.

- **Design intuitions**
  - **Residual connection** mimics forward Euler scheme;
  - **Physics-based Conv layer** encodes existing terms in $\mathcal{F}$;
  - **Element-wise product layer** approximates nonlinears terms better.

Π Elementwise product layer

Boundary Encoding



- **BCs (if available) are forcibly encoded to ensure solution accuracy**

Element-wise Product Layer

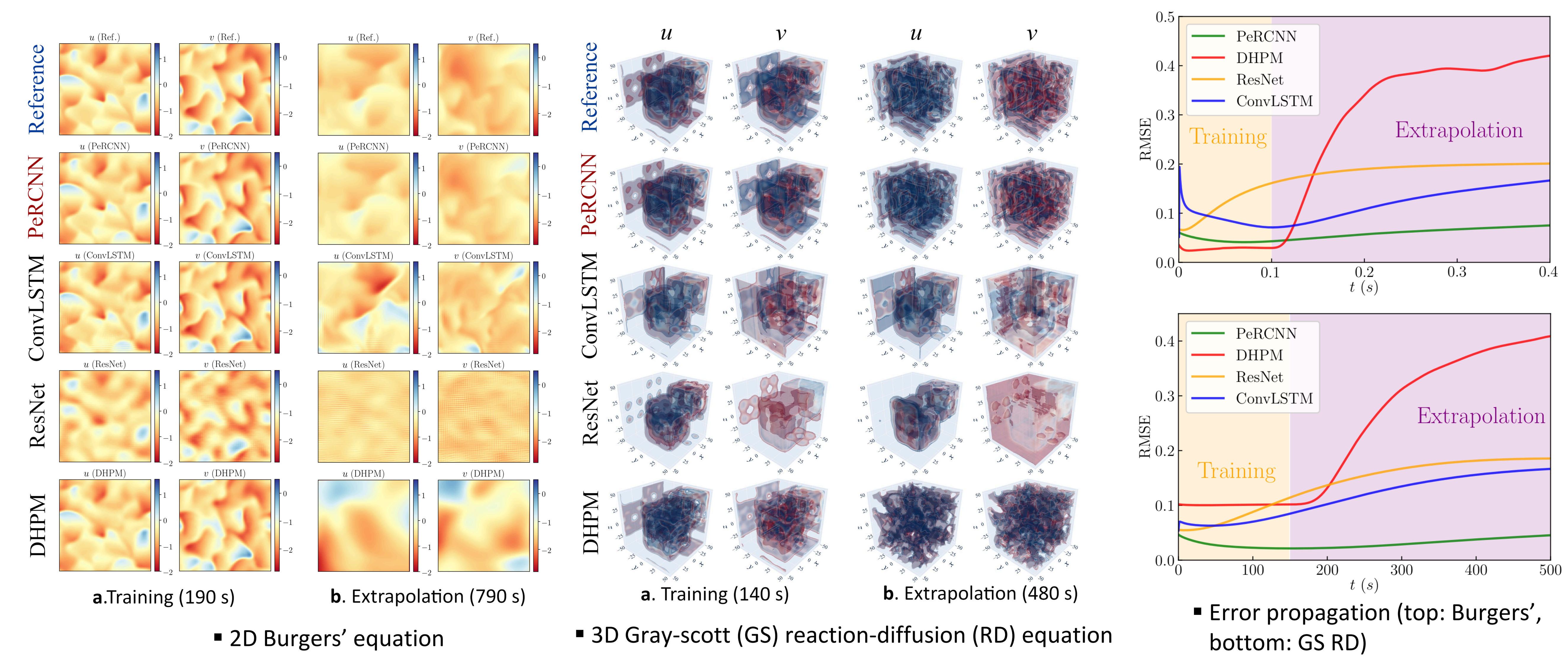
- **Π-block approximation**

$$\mathcal{F}(\boldsymbol{u}) = \sum_{c=1}^{N_c} f_c \left(\prod_{l=1}^{N_l} D^{(c,l)} \circledast \boldsymbol{u} \right)$$

- **Benefits**
  - Multiplicative form makes the learned model more interpretable;
  - Enables a better approximation to nonlinear terms like uu_x and $\mathbf{u}\cdot\nabla u$.

Numerical Experiments

- **Baselines:** ConvLSTM, Deep Hidden Physics Model (DHPM) and Recurrent ResNet



- **PeRCNN outperforms the baselines on accuracy;**
- **PeRCNN generalizes well beyond the training region where no data is available.**

References

[1] Raissi, M. (2018). Deep hidden physics models: Deep learning of nonlinear partial differential equations. The Journal of Machine Learning Research, 19(1), 932-955.
[2] Shi, X., Chen, Z., Wang, H., Yeung, D. Y., Wong, W. K., & Woo, W. C. (2015). Convolutional LSTM network: A machine learning approach for precipitation nowcasting. arXiv preprint arXiv:1506.04214.
[3] He, K., Zhang, X., Ren, S., & Sun, J. (2016). Deep residual learning for image recognition. In Proceedings of the IEEE conference on computer vision and pattern recognition (pp. 770-778).