

CosmicRIM : Reconstructing Early Universe

by Combining Differentiable Simulations with Recurrent Inference Machines

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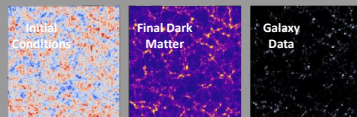
Motivation

Aim : To reconstruct the **Gaussian** initial conditions^[1] at the beginning of the Universe from a sparse galaxy sample data

Challenge :

1. solve an inverse problem in high-dimensions ($\sim 10^8$) to reconstruct density at all points in space
2. forward model is complex, *non-linear* and expensive N-body simulations

Approach : Combine *differentiable* cosmological N-body simulations, like **FlowPM**^[2] with *learnt optimization* methods to learn inference schemes^[3] and tackle these challenges efficiently.



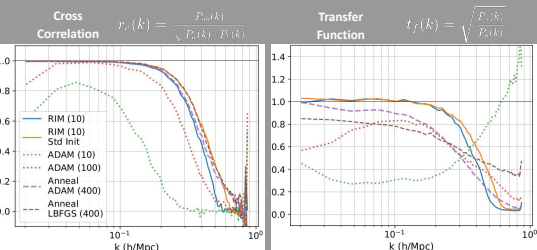
Experiments

Forward model : 64^3 particles, 2 step PM sim. with 2nd order bias model

CosmicRIM outperforms traditional optimization, even with *physically* motivated annealing at **40x** less cost

Informed initial position^[5] \mathbf{x}_0 can improve reconstruction (see orange)

Comparing 2-point statistics between reconstructed & true initial conditions



CosmicRIM

Setup : To obtain the MAP estimate given the likelihood p of the observed data \mathbf{y} for the initial conditions \mathbf{x} and Gaussian prior p_ϕ on the initial conditions with known power spectrum

$$\max_{\mathbf{x}} \ln p(\mathbf{x}, \mathbf{y}) = \max_{\mathbf{x}} [\ln p(\mathbf{y}|\mathbf{x}) + \ln p_\phi(\mathbf{x})]$$

RIM (Recurrent inference machine)^[4] : learn optimization by training a recurrent neural network (h_ϕ, g_ϕ) to learn the update equations at time step t , given state \mathbf{s}_t

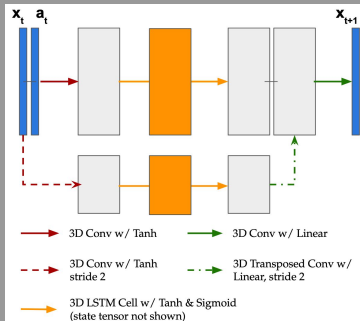
$$\begin{aligned} \mathbf{s}_{t+1} &= \mathbf{s}_t + h_\phi(\mathbf{s}_t, \nabla_{\mathbf{x}} \ln p(\mathbf{x}, \mathbf{y}), \mathbf{x}_t), \\ \mathbf{x}_{t+1} &= \mathbf{x}_t + g_\phi(\mathbf{x}_t, \nabla_{\mathbf{x}} \ln p(\mathbf{x}, \mathbf{y}), \mathbf{s}_{t+1}) \end{aligned}$$

Training loss function for a 10 step RIM

$$\mathcal{L} = \sum_{t=0}^{10} (\mathbf{x}_t(\phi) - \mathbf{x}_{true})^2$$

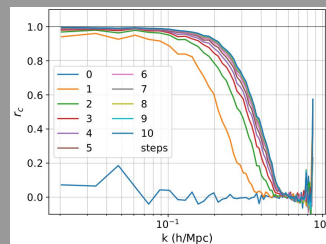
Architectural modifications

1. **Replace gradients** $\nabla_{\mathbf{x}} \ln p(\mathbf{x}, \mathbf{y})$ in the update functions (h, g) with update predicted by **ADAM** algorithm \mathbf{a}_t
2. **U-Net** architecture with different LSTM cells in **parallel** to learn updates for large and small scales separately



What CosmicRIM learns?

Physically motivated annealing : large scales have higher SNR & linear dynamics
This led [6] to develop annealing scheme that smooth the likelihood on small scales & reconstruct the large scales first.
RIM implicitly learns a similar path to reconstruction



(Local) Minima : i.e ADAM/L-BFGS starting from RIM output don't improve results

Outlook

- **First application** combining complex, non-linear albeit differentiable forward models with learnt optimization schemes for high-dimensional inference problems.
- **40x speed up** & better reconstruction of the initial conditions of the Universe with CosmicRIM over other approaches
- **Implicitly learning** the optimization path otherwise strictly imposed with physically motivated annealing schemes.
- **Challenge** : High memory requirements for training can be a bottleneck but potential solutions can be found in splitting the optimization path and sequential learning

References

- [1] U. Seljak, G. Aslanyan, Y. Feng & C. Modi. arXiv:1706.06645
- [2] M Andrychowicz et al. arXiv:1606.0447
- [3] C. Modi, F. Lanusse, U. Seljak. (*FlowPM*) arXiv:2010.11847
- [4] P. Putzky & M. Welling. (*RIM*) arXiv:1706.04008
- [5] D Eisenstein et al.(Std. Reconstruction) ApJ,664(2):675–679
- [6] C. Modi, Y. Feng, U. Seljak. arXiv: 1805.02247