Numerical simulators require substantial efforts to build. Different models needed to be built for different scenarios. High fidelity simulation based on numerical models demands substantial computational resources. We propose a graph-based data-driven model for Lagrangian fluid simulation, Fluid Graph Networks (FGN), which consists of multi-layer perceptron and graph inductive architectures. The trained model is ~10 to 100 times faster than the numerical solver.

**Motivation**
- Numerical simulators require substantial efforts to build. Different models needed to be built for different scenarios.
- High fidelity simulation based on numerical models demands substantial computational resources.

We use graph to describe the fluid field, where fluid particles are defined as nodes in the graph. Two types of graph neural networks are proposed to emulate the different physical processes involved in fluid dynamics. They can be divided into two types of graph networks (GN) according to the prediction tasks.

- **Node-focused Network**
  - We use node-focused GN to predict advection and projection process in fluid simulation. Advection net is responsible for the prediction of advection effect and pressure net is responsible for projection. In node-focused network, the final output of the network is a set of node features (e.g. particle’s acceleration, particle’s pressure).

  For a specific layer of node-focused network, the update rule is defined as:
  \[
  \mathbf{a}^{(l)}_i = \frac{\sum_{j \in \mathcal{N}(i)} W(d_i - d_j, h) + \mathbf{f}^{(l-1)}_j}{\sum_{j \in \mathcal{N}(i)} W(d_i - d_j, h)} + \mathbf{f}^{(l-1)}_j, \quad \forall j \in \mathcal{N}(i).
  \]

- **Edge-focused Network**
  - We use edge-focused GN to drive away particles that are too close to each other, which emulates the elastic collision process. In edge-focused network, the input of the network is a set of edge features, and in the last layer all edge embeddings are aggregated to nodes.

  The update rule of edge-focused network is defined as:
  \[
  r_{ij} = \sigma(e_{ij}),
  \]

  \[
  a_i = \sum_{j \in \mathcal{N}(i)} W(r_{ij})
  \]

**Method**

**Visualization**

- **Extrapolation to complex geometries**
- **Model predicts reasonable pressure distribution**

**Results**

- **Quantitative comparison with Ground Truth on test case**
  - **Case** | **Model** | **Density error** | **Velocity divergence** | **Average Chamfer distance (mm)**
  - **Dam Collapse**
    - FGN (this work) | 0.0461 | 0.0900 | 0.0195 | 0.0280 | 24.2 | 30.1
    - Ground Truth | 0.0380 | 0.0710 | 0.0190 | 0.0268 | - | -
  - **Water Fall**
    - FGN (this work) | 0.0841 | 0.1300 | 0.0207 | 0.0431 | 24.8 | 29.6
    - Ground Truth | 0.0429 | 0.0966 | 0.0196 | 0.0398 | - | -

- **Evaluation of sub-networks**

<table>
<thead>
<tr>
<th>Model</th>
<th>Dynamical system</th>
<th>Model Output</th>
<th>Evaluation</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advection Net</td>
<td>( \dot{u} = g + \nabla p \cdot u )</td>
<td>prediction of ( \dot{u} )</td>
<td>Normalized MAE</td>
<td>12.4% ± 5.6%</td>
</tr>
<tr>
<td>Pressure Net</td>
<td>( \nabla p = \frac{1}{\rho} \nabla \cdot \tau )</td>
<td>prediction of ( p )</td>
<td>Relative tolerance</td>
<td>8.3% ± 1.1%</td>
</tr>
</tbody>
</table>

- **Runtime analysis**
  - Total trainable parameters: 41996, can be trained within 2 hrs (1M iterations) on a single GPU.
  - Single frame inference time is ~40 ms for a 10000 particles scene, which is ~15 times faster than baseline solver (Moving Particle Semi-implicit method).

**Conclusion**

A fast data-driven model for particle-based fluid simulation is proposed, which can greatly improve calculation efficiency without compromising much accuracy and stability.