Online parameter inference for the simulation of a Bunsen flame using heteroscedastic Bayesian neural network ensembles

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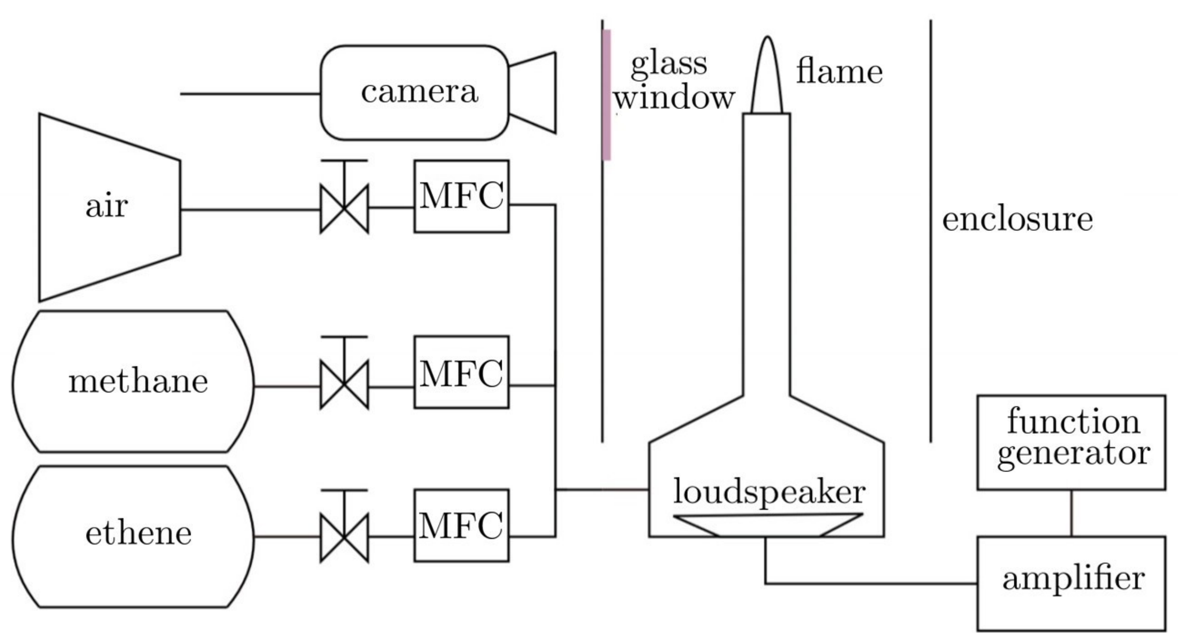
1. Introduction

- Task:

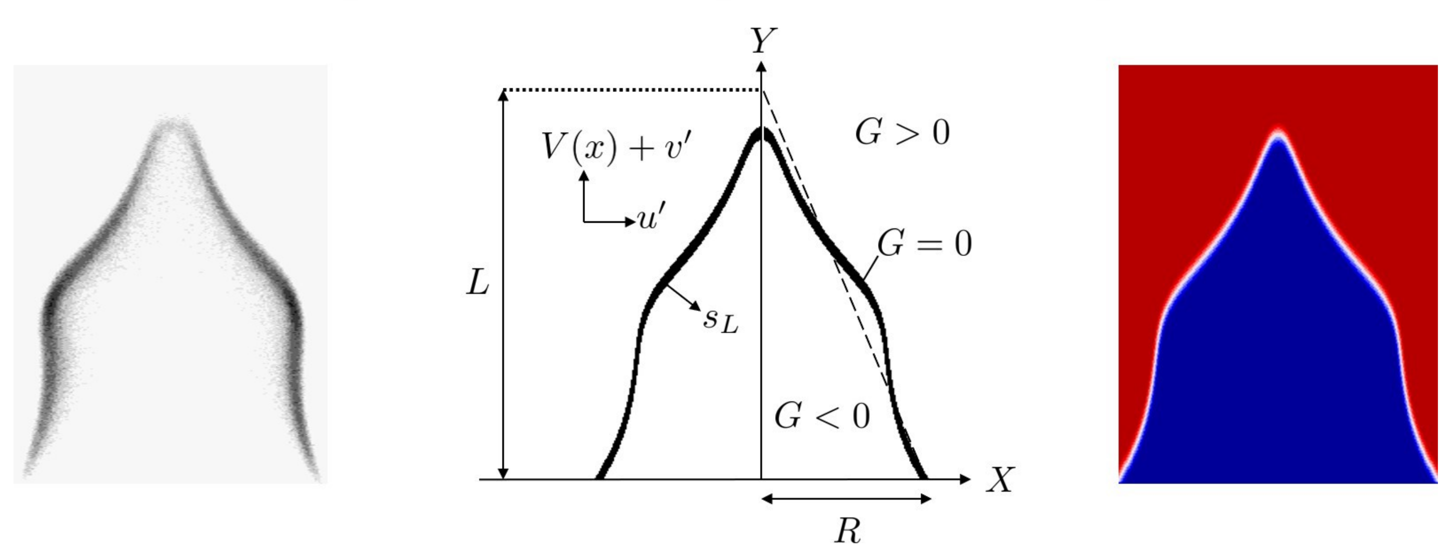
  "Can we infer the parameters of the model from images of the flame?"

- This would allow us to **predict future behaviour** and calculate key quantities like the **heat release rate**



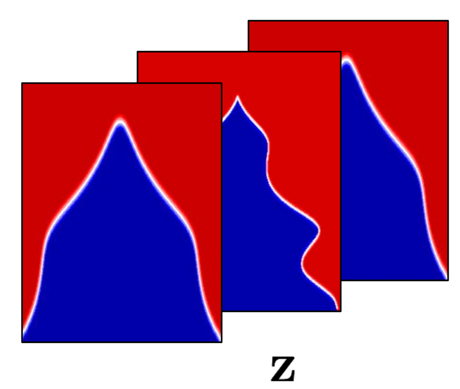
2. Flame model and simulator

- The flame is modelled as a thin boundary between reactants and products using the G-equation (Williams 1985)
- The **parameters of the model** include flame dimensions, perturbation speeds and frequencies



3. Bayesian neural network ensemble

- Training: learn the parameters from a **library of flame shapes** generated with the simulator

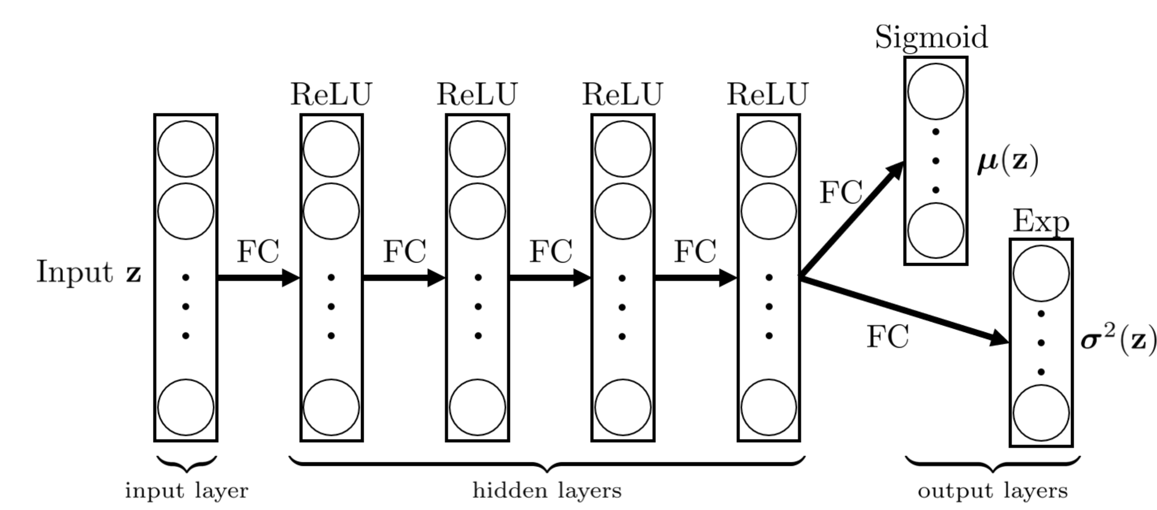


For known parameters, t learn means and variances: $\mu_j(\mathbf{z}),\ \sigma_j^2(\mathbf{z})$ assuming the parameters are Gaussian distributed: $\mathbf{t} \sim \mathcal{N}\left(\mathbf{t};\ \mu_j(\mathbf{z}),\ \sigma_j^2(\mathbf{z})\right)$

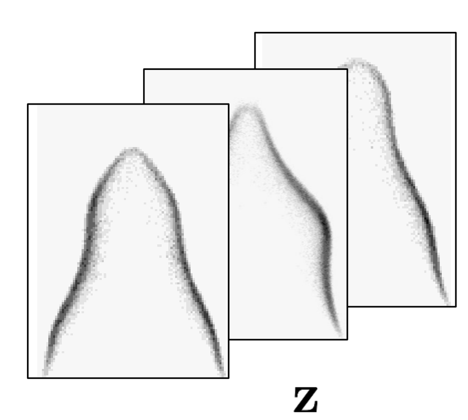
- Loss function**: maximise the Bayesian posterior** of the parameters given the flame shapes

$$\mathcal{L}_j = \underbrace{\left(\mu_j(\mathbf{z})-\mathbf{t}\right)^T \Sigma_j(\mathbf{z})^{-1}\left(\mu_j(\mathbf{z})-\mathbf{t}\right) + \log\left(\left|\Sigma_j(\mathbf{z})\right|\right)}_{\text{Gaussian likelihood}} + \underbrace{\left(\theta_j-\theta_{anc,j}\right)^T \Sigma_{prior}^{-1}\left(\theta_j-\theta_{anc,j}\right)}_{\text{anchored prior}}$$

- Neural network architecture (20 in the ensemble)

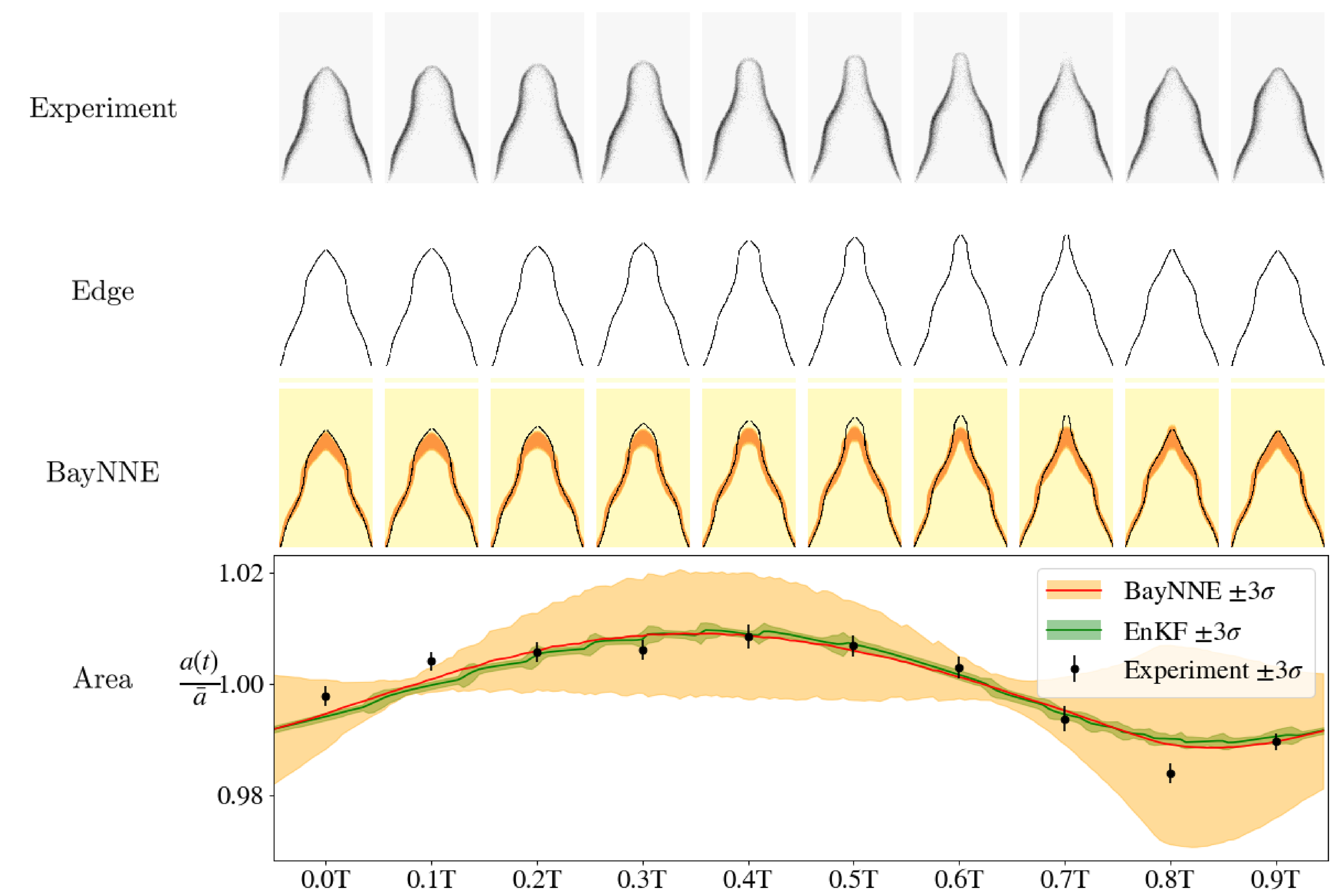


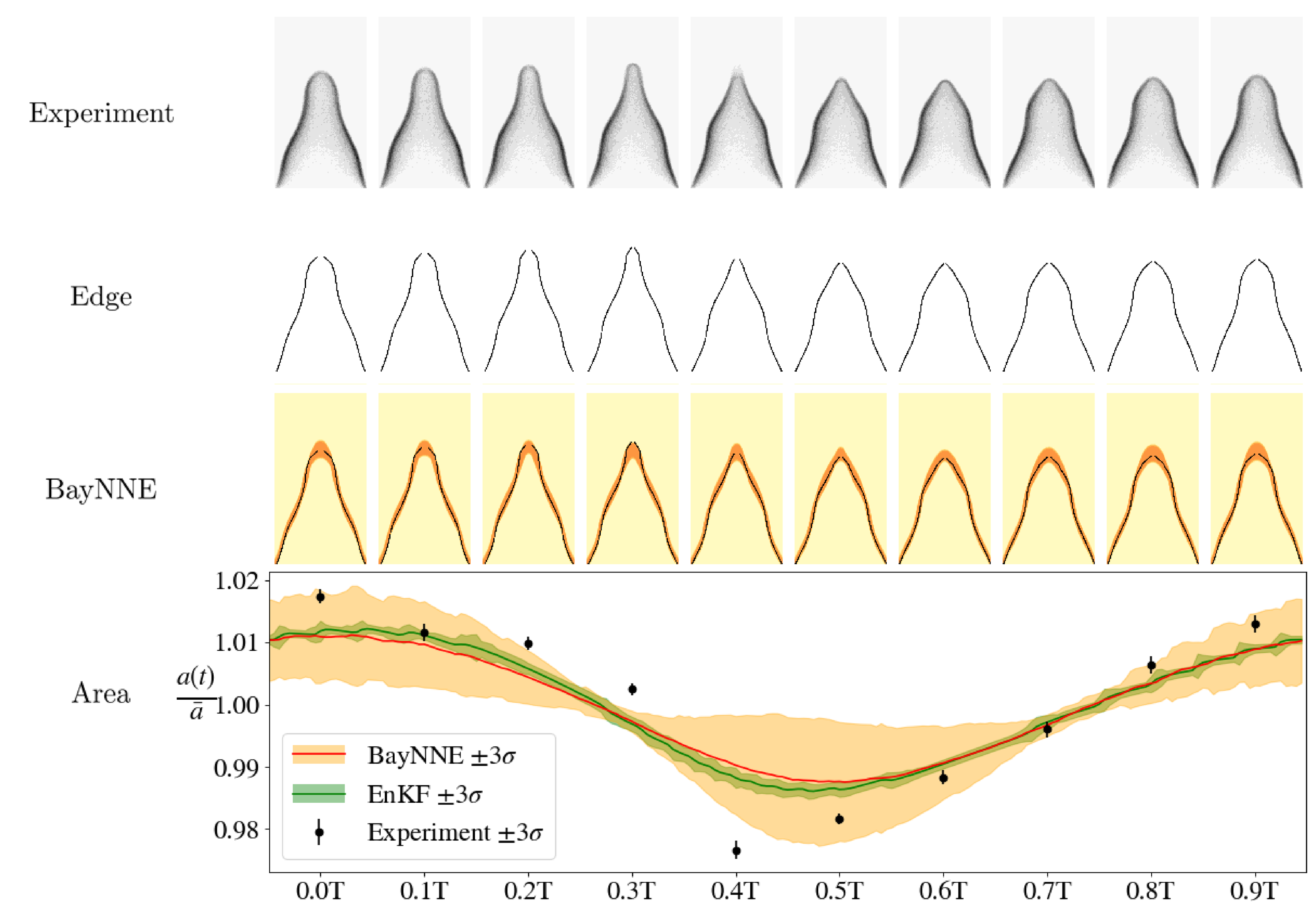
- Testing: infer the parameters from the experiments



$$\mu_j(\mathbf{z}),\ \ \sigma_j^2(\mathbf{z})$$

4. Results





5. Summary

The BayNNE:

- is trained on a library of simulated flame shapes and tested on real data
- **accurately infers parameters of the flame model orders of magnitude faster than the state-of-the-art**