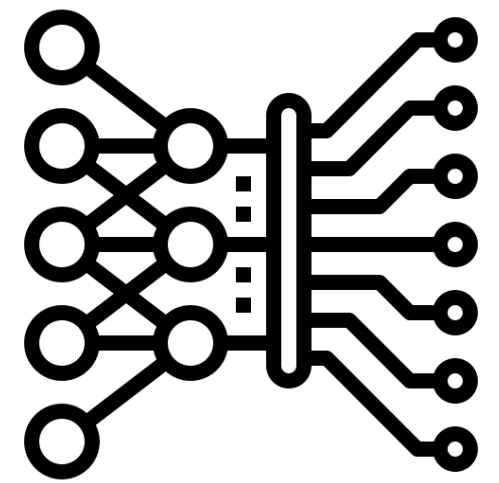




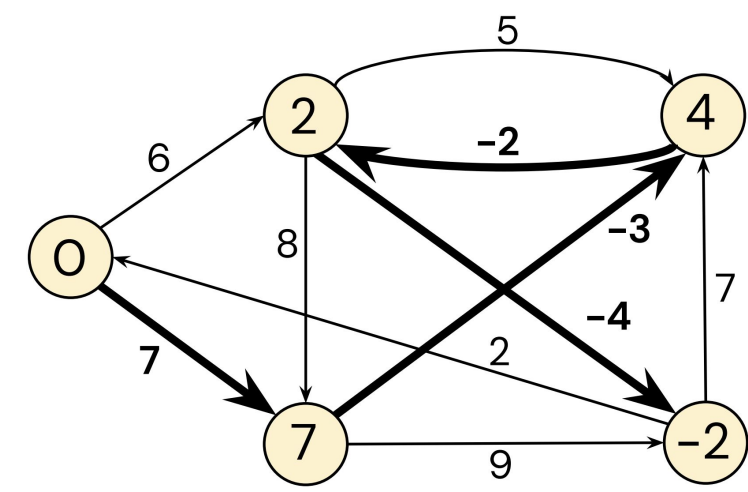
Persistent Message Passing

Heiko Strathmann, Mohammadamin Barekatin, Charles Blundell, Petar Veličković

Algorithmic Reasoning with GNNs



Neural networks



Algorithms

- + Operate on **raw** inputs
- + Generalise on **noisy** conditions
- + Models **reusable** across tasks
- Require **big data**
- Unreliable when **extrapolating**
- Lack of **interpretability**

- + Trivially **strongly** generalise
- + **Compositional** (subroutines)
- + Guaranteed **correctness**
- + **Interpretable** operations
- Inputs must match **spec**
- Not **robust** to task variations

Non-Markovian Inputs

GNNs typically overwrite their internal states in every step

- Previous applications: only latest version of the data relevant
- Here: queries require knowledge of previous versions of the data
- This places significant "pressure" on internal states

Current GNNs cannot explicitly memorize their past states

Persistent Message Passing

Idea: endow GNNs with an explicit, **persistent**, memory

- Instead of overwriting nodes -> persisting nodes
- Paired with an efficient relevance / query mechanism
- Effectively an **episodic memory** of past computation

Strong performance on non-Markovian tasks

Input Encoding

Initial connectivity

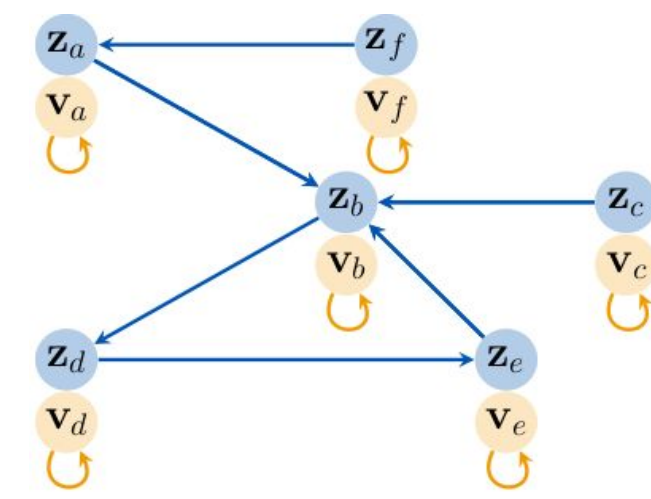
$$\Pi^{(0)} = \Pi$$

$$\Lambda^{(0)} = I$$

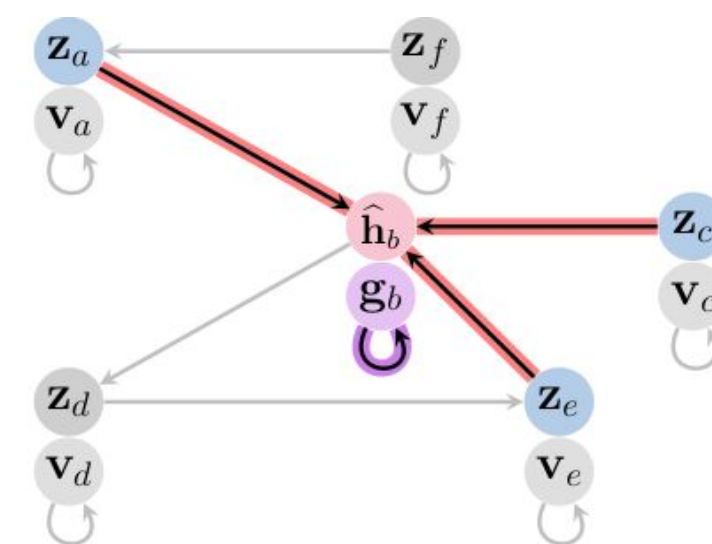
Node representations

$$\mathbf{v}_j^{(t)} = f_{\text{relevance}}(\text{time_stamp}(j), t, \mathbf{h}_j^{(t-1)})$$

$$\mathbf{z}_j^{(t)} = f_{\text{operation}}(\text{expand}(E^{(t)}, s^{(t)}), \mathbf{h}_j^{(t-1)})$$



Messages



Next-step candidates

$$\hat{\mathbf{h}}_j^{(t)} = U \left(\mathbf{z}_j^{(t)}, \max_{\Pi_{j,j'}^{(t-1)}=1} M(\mathbf{z}_{j'}^{(t)}, \mathbf{z}_j^{(t)}) \right)$$

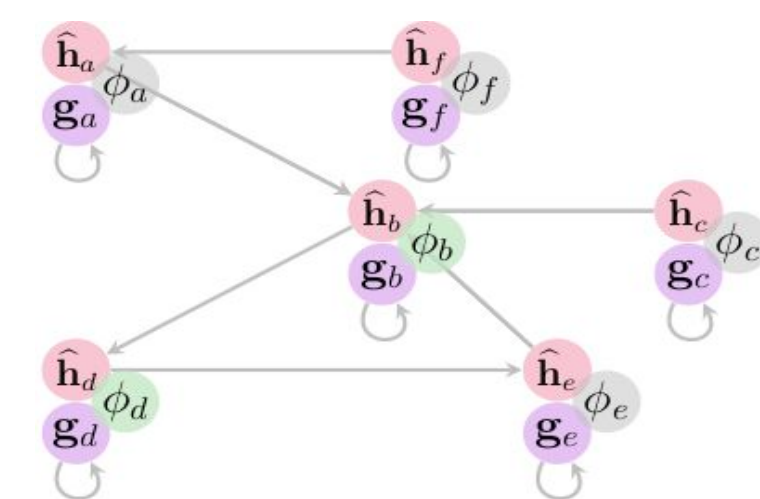
Relevance latents

$$\mathbf{g}_j^{(t)} = U \left(\mathbf{v}_j^{(t)}, \max_{\Lambda_{j,j'}^{(t-1)}=1} M(\mathbf{v}_{j'}^{(t)}, \mathbf{v}_j^{(t)}) \right)$$

Masking

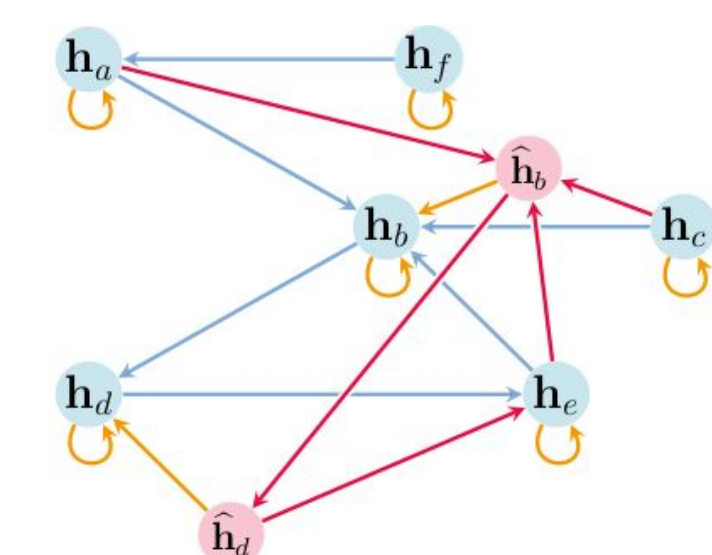
$$\mu_j^{(t)} = \psi_{\text{relevance}}(\mathbf{g}_j^{(t)}) \in \{0, 1\}$$

$$\phi_j^{(t)} = \psi_{\text{persistence}}(\hat{\mathbf{h}}_j^{(t)} \cdot \mu_j^{(t)}) \in \{0, 1\}$$



Persistency

- "Persist" selected nodes
- Original nodes untouched
- Track connectivity

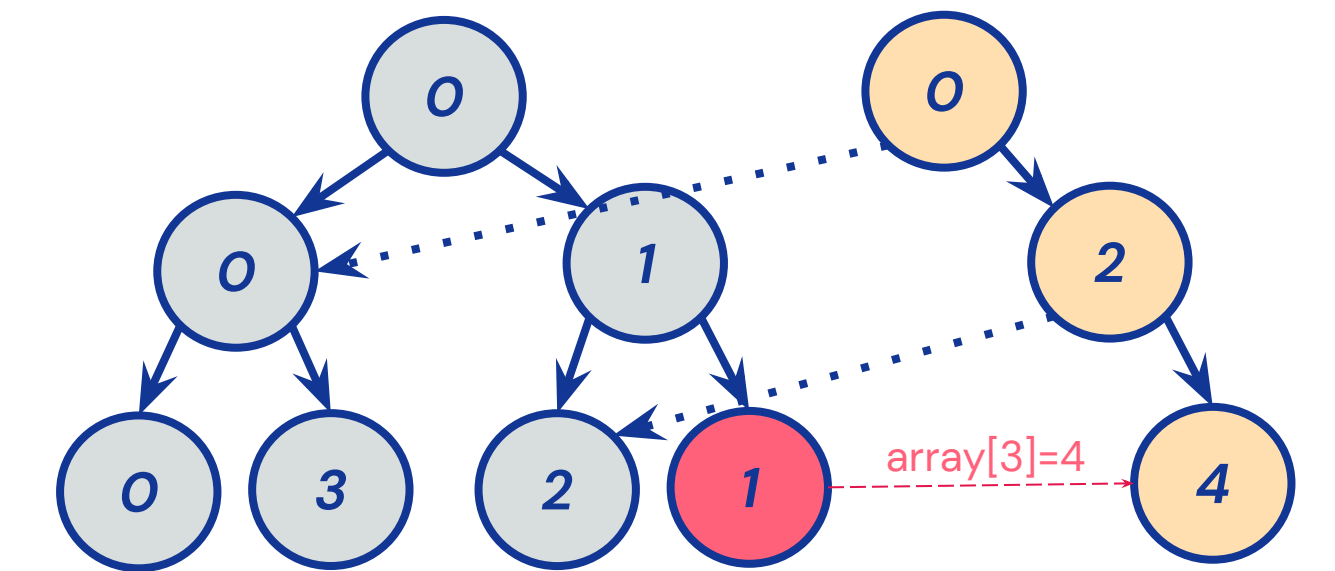


Task: Dynamic Range Minimum Queries

Compute (historic) minimum across ranges in continuously updated array

Solved by Segment Trees

- $O(\log n)$ query time
- Leaf nodes store array
- Intermediate nodes store minimum of children



Results

- PMP generalizes out-of-distribution to 2x larger test inputs
- Significantly outperforms GNNs which overwrite states
- Supervised on response (binary encoding), ground-truth masks, and node-values

