

A EXPERIMENTAL DETAILS

Table 4: Architecture optimizer settings on PDE tasks with CNN and FNO initializations. Note that the learning rate is updated using the same schedule as the backbone.

| task | XD init | optimizer | initial lr | warmup |
|--|---------|---------------|------------|--------|
| 1d Burgers' equation | CNN | Adam | 1E-3 | 0 |
| 1d Burgers' equation | FNO | Momentum(0.5) | 1E-4 | 250 |
| 2d Darcy Flow | CNN | Momentum(0.5) | 1E-1 | 0 |
| 2d Darcy Flow | FNO | Momentum(0.5) | 1E-1 | 0 |
| 2d Navier Stokes ($\nu = 10^{-4}, T = 30$) | CNN | Momentum(0.5) | 5E-3 | 0 |
| 2d Navier Stokes ($\nu = 10^{-5}, T = 20$) | CNN | Momentum(0.5) | 1E-3 | 0 |

For our experiments, we use the code and setup from Li et al. (2021b) provided here: https://github.com/zongyi-li/fourier_neural_operator. We use the same training routine and settings as the backbone architecture for each task and only tune the architecture optimizer. We consider the following hyperparameters for the architecture optimizer: Adam vs. SGD (with or without momentum), initial learning rate, and number of warmup epochs. Our CNN backbone is analogous to the FNO architecture used for each problem. In particular, the CNN backbone architecture used for each task is simply the FNO architecture where FNO layers of dimension N with m modes are replaced by N -dimensional convolutional layers with filters of size $(m + 1)^N$ and circular padding to match the dimensionality of FNO.

Table 5: Test relative errors on the 1d Burgers' equation. We were not able to match the FNO-1d results reported by Li et al. (2021b) using their published codebase, however, our proposed XD operations outperform our reproduction of their results at every resolution. Furthermore, we outperform their reported test relative errors on every resolution except $s = 4096$, where we roughly match their performance.

| Method (source) | $s = 256$ | 512 | 1024 | 2048 | 4096 | 8192 |
|---------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| NN (Li et al., 2021b) | 0.4714 | 0.4561 | 0.4803 | 0.4645 | 0.4779 | 0.4452 |
| GCN (Li et al., 2021b) | 0.3999 | 0.4138 | 0.4176 | 0.4157 | 0.4191 | 0.4198 |
| FCN (Li et al., 2021b) | 0.0958 | 0.1407 | 0.1877 | 0.2313 | 0.2855 | 0.3238 |
| PCANN (Li et al., 2021b) | 0.0398 | 0.0395 | 0.0391 | 0.0383 | 0.0392 | 0.0393 |
| GNO (Li et al., 2021b) | 0.0555 | 0.0594 | 0.0651 | 0.0663 | 0.0666 | 0.0699 |
| LNO (Li et al., 2021b) | 0.0212 | 0.0221 | 0.0217 | 0.0219 | 0.0200 | 0.0189 |
| MGNO (Li et al., 2021b) | 0.0243 | 0.0355 | 0.0374 | 0.0360 | 0.0364 | 0.0364 |
| FNO-1d (Li et al., 2021b) | 0.0149 | 0.0158 | 0.0160 | 0.0146 | 0.0142 | 0.0139 |
| CNN backbone (ours) | 0.0518 | 0.1220 | 0.1830 | 0.2280 | 0.2730 | 0.2970 |
| FNO-1d (reproduced) | 0.0181 | 0.0191 | 0.0188 | 0.0184 | 0.0183 | 0.0183 |
| CNN backbone XD (ours) | 0.0141 | 0.0079 | 0.0154 | 0.0099 | 0.0145 | 0.0123 |
| FNO-1d XD (ours) | 0.0153 | 0.0154 | 0.0154 | 0.0167 | 0.0160 | 0.0155 |

Table 6: Test relative errors on 2d Darcy Flow. Our reproduction of the FNO-2d results outperform those reported by Li et al. (2021b). Nonetheless, our proposed XD operations outperform both our reproduction and the reported results of Li et al. (2021b) at every resolution.

| Method (source) | $s = 85$ | $s = 106$ | $s = 141$ | $s = 211$ | $s = 421$ |
|---------------------------|---------------|---------------|---------------|---------------|---------------|
| NN (Li et al., 2021b) | 0.1716 | - | 0.1716 | 0.1716 | 0.1716 |
| GCN (Li et al., 2021b) | 0.0253 | - | 0.0493 | 0.0727 | 0.1097 |
| FCN (Li et al., 2021b) | 0.0299 | - | 0.0298 | 0.0298 | 0.0299 |
| PCANN (Li et al., 2021b) | 0.0244 | - | 0.0251 | 0.0255 | 0.0259 |
| GNO (Li et al., 2021b) | 0.0346 | - | 0.0332 | 0.0342 | 0.0369 |
| LNO (Li et al., 2021b) | 0.0520 | - | 0.0461 | 0.0445 | - |
| MGNO (Li et al., 2021b) | 0.0416 | - | 0.0428 | 0.0428 | 0.0420 |
| FNO-2d (Li et al., 2021b) | 0.0108 | - | 0.0109 | 0.0109 | 0.0098 |
| CNN backbone (ours) | 0.0404 | 0.0495 | 0.0613 | 0.0813 | 0.1150 |
| FNO-2d (reproduced) | 0.0096 | 0.0092 | 0.0091 | 0.0091 | 0.0091 |
| CNN backbone XD (ours) | 0.0065 | 0.0065 | 0.0065 | 0.0071 | 0.0066 |
| FNO-2d XD (ours) | 0.0082 | 0.0079 | 0.0077 | 0.0076 | - |